

Stocksbridge Junior School



Learning Together

Calculation Guide for Parents

Year 4

Introduction

At the centre of the mastery approach to the teaching of Mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This document outlines the different calculation strategies that should be taught and used in Years 1 to 6, in line with the requirements of the 2014 Primary National Curriculum.

Purpose

The purpose of this document is threefold. Firstly, it outlines the structures for calculations, which ensure pupils are exposed to both standard and non-standard problems. Secondly, it makes parents aware of the strategies that pupils are formally taught within each year group, which will support them to perform mental and written calculations. Finally, it supports teachers and parents in identifying appropriate pictorial representations and concrete materials to help develop understanding.

How to use the document

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The purpose of using multiple representations is to give pupils a deep understanding of a mathematical concept and they should be able to work with and explain concrete, pictorial and abstract representations, and explain the links between different representations. Depth of understanding is achieved by moving between these representations. For example, if a child has started to use a pictorial representation, it does not mean that the concrete cannot be used alongside the pictorial. If a child is working in the abstract, depth can be evidenced by asking them to exemplify their abstract working using a concrete or pictorial representation and to explain what they have done using the correct mathematical vocabulary.

Progression in calculations

Year 4

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

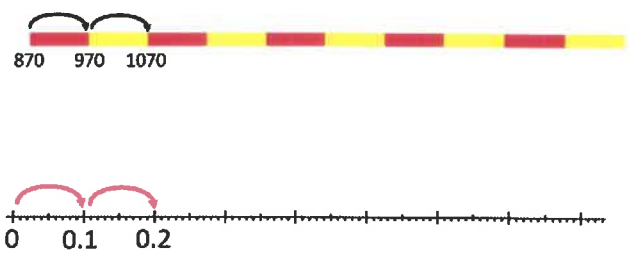
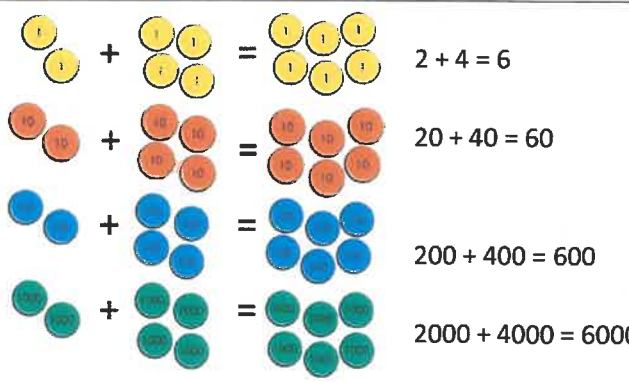
- add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
- find 1 000 more or less than a given number
- estimate and use inverse operations to check answers to a calculation

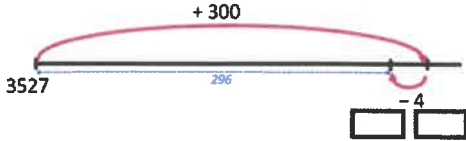
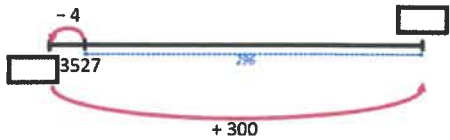
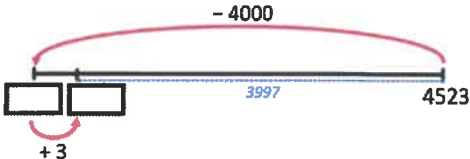
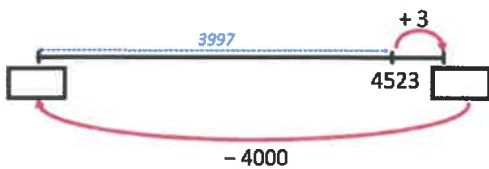
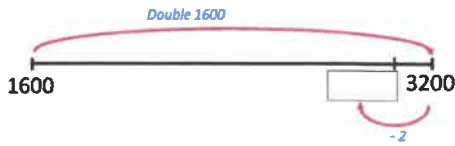
N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

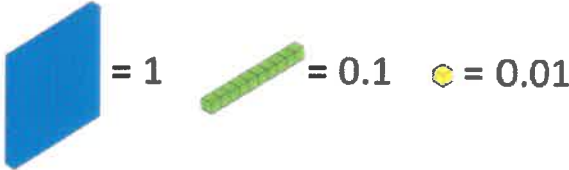


















- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving fractions and decimals to two decimal places

Y4 Addition & Subtraction

Strategies & Guidance	CPA
<p>Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000.</p> <p><i>Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.</i></p> <p>Count forwards and backwards in tenths and hundredths</p>	 <p>Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.</p> <p>E.g. $990 + 10$ or $19.9 + 0.1$</p>
<p>Using known facts and knowledge of place value to derive facts.</p> <p>Add and subtract multiples of 10, 100 and 1000 mentally</p> <p><i>Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.</i></p>	 <p>$2 + 4 = 6$</p> <p>$20 + 40 = 60$</p> <p>$200 + 400 = 600$</p> <p>$2000 + 4000 = 6000$</p>
<p>Adding and subtracting by partitioning one number and applying known facts.</p> <p><i>By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.</i></p>	<p>See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.</p>

Strategies & Guidance	CPA
<p>Round and adjust</p> <p><i>Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.</i></p> <p><i>It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.</i></p> <p><i>Build flexibility by completing the same calculation in a different order.</i></p>	<p>$3527 + 296 = 3827 - 4$</p>  <p>Completing the same calculation but adjusting first:</p> <p>$3527 + 296 = 3523 + 300$</p>  <p>$4523 - 3997 = 523 + 3$</p>  <p>Completing the same calculation but adjusting first:</p> <p>$4523 - 3997 = 4526 - 4000$</p> 
<p>Near doubles</p> <p><i>Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.</i></p>	<p>$1600 + 1598 = \text{double } 1600 - 2$</p> 

Strategies & Guidance	CPA																																																																				
<p>Written column methods for addition</p> <p><i>Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.</i></p> <p><i>This method and the language to use are best understood through the tutorial videos found here on the toolkit.</i></p>	<div style="display: flex; flex-direction: column; align-items: center;"> <table border="1" style="margin-bottom: 10px;"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3 green</td> <td>2 blue</td> <td>7 orange</td> <td>3 yellow</td> </tr> <tr> <td></td> <td>4 blue</td> <td>4 orange</td> <td>1 yellow</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <table border="1" style="margin-bottom: 10px;"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3 green</td> <td>2 blue</td> <td></td> <td></td> </tr> <tr> <td></td> <td>4 blue</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>10 orange</td> <td>4 yellow</td> </tr> <tr> <td></td> <td></td> <td>1 orange</td> <td></td> </tr> </tbody> </table> <table border="1" style="margin-bottom: 10px;"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>3 green</td> <td>4 blue</td> <td>1 orange</td> <td>4 yellow</td> </tr> </tbody> </table> <div style="margin-left: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>5</td><td>2</td><td>7</td><td>3</td></tr> <tr><td>+</td><td>5</td><td>4</td><td>1</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td>5</td><td>8</td><td>1</td><td>4</td></tr> </table> </div> </div>	Thousands	Hundreds	Tens	Ones	3 green	2 blue	7 orange	3 yellow		4 blue	4 orange	1 yellow					Thousands	Hundreds	Tens	Ones	3 green	2 blue				4 blue					10 orange	4 yellow			1 orange		Thousands	Hundreds	Tens	Ones									3 green	4 blue	1 orange	4 yellow	5	2	7	3	+	5	4	1	<hr/>				5	8	1	4
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Strategies & Guidance	CPA												
<p>Calculating with decimal numbers</p> <p><i>Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.</i></p> <p><i>Place value counters are another useful manipulative for representing decimal numbers.</i></p> <p><i>All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.</i></p>	<p style="text-align: center;">CPA</p> <div style="text-align: center;">  <p>= 1 = 0.1 = 0.01</p> </div> <p>$24.2 + 13.4 =$</p> <table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th style="width: 33%;">Tens</th> <th style="width: 33%;">Ones</th> <th style="width: 33%;">Tenths</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td> </td> <td> </td> <td> </td> </tr> </tbody> </table>	Tens	Ones	Tenths									
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National Curriculum objectives linked to multiplication and division

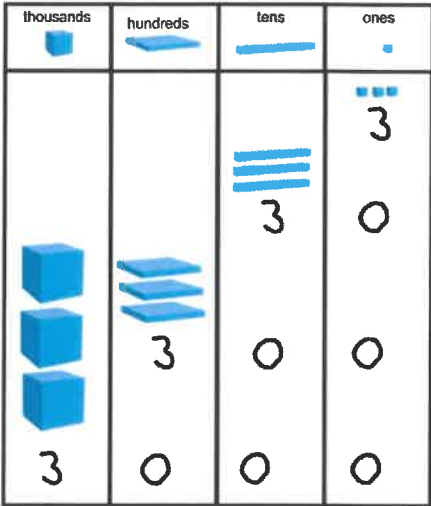
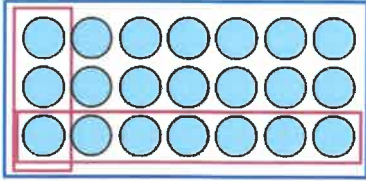




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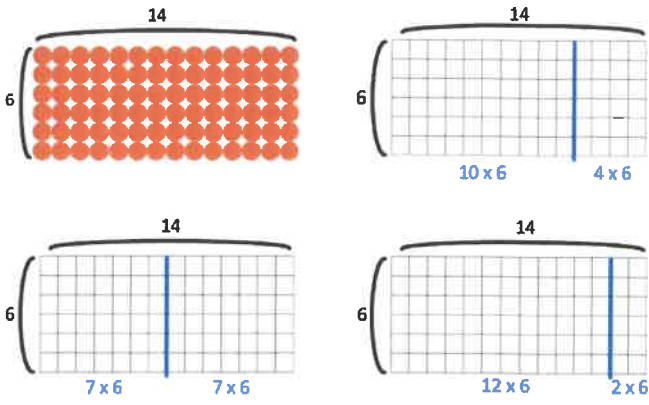
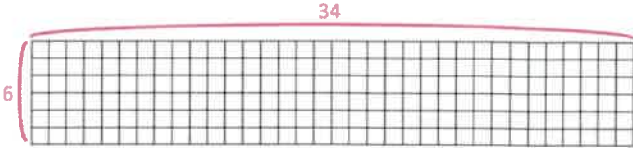
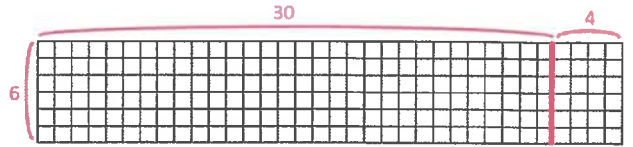

- count from 0 in multiples of 6, 7, 9, 25 and 1000
- recall and use multiplication and division facts for multiplication tables up to 12×12
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- recognise and use factor pairs and commutativity in mental calculations
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

























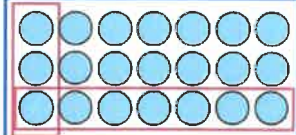
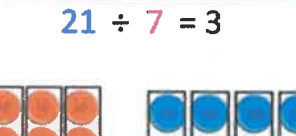


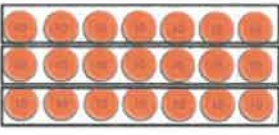
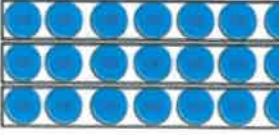
Y4 Multiplication

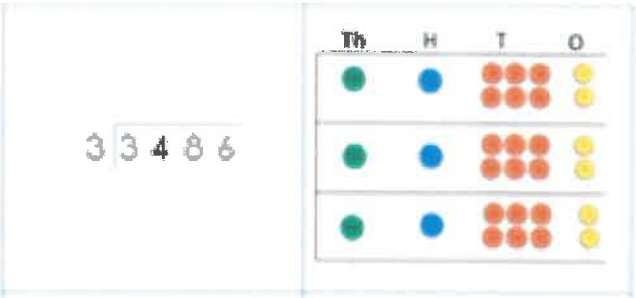
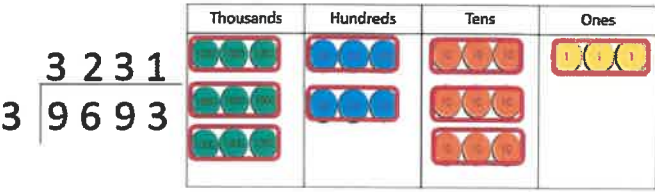
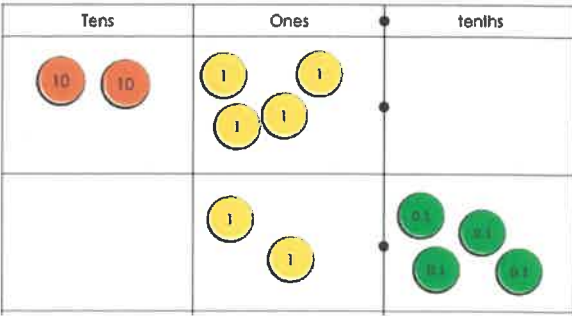
Strategies & Guidance	CPA
<p>Multiplying by 10 and 100</p> <p><i>When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.</i></p> <p><i>When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.</i></p> <p><i>Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000</i></p>	 <p>$3 \times 10 = 30$</p> <p>$3 \times 100 = 300$</p> <p>$3 \times 1000 = 3000$</p>
<p>Using known facts and place value for mental multiplication involving multiples of 10 and 100</p> <p><i>Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.</i></p> <p><i>Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing for larger 'fact families' to be derived from a single known number fact.</i></p> <p><i>Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.</i></p>	<p>factor factor product $3 \times 7 = 21$</p>  <p>factor factor product $7 \times 3 = 21$</p>     <p>$30 \times 7 = 210$ $300 \times 7 = 2100$</p> <p>$70 \times 3 = 210$ $700 \times 3 = 2100$</p> <p>$7 \times 30 = 210$ $7 \times 300 = 2100$</p> <p>$3 \times 70 = 210$ $3 \times 700 = 2100$</p>

Strategies & Guidance	CPA
<p>Multiplying by partitioning one number and multiplying each part</p> <p><i>Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.</i></p> <p><i>As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.</i></p> <p>Distributive law</p> <p>$a \times (b + c) = a \times b + a \times c$</p>	<p>14×6</p>  <p>34×6</p>  <p>$30 \times 6 + 4 \times 6$</p> 
<p>Mental multiplication of three 1-digit numbers, using the associative law</p> <p><i>Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.</i></p>	<p>Four pots each containing two flowers which each have seven petals. How many petals in total?</p>  <p>$(4 \times 2) \times 7$ or $4 \times (2 \times 7)$</p>

Strategies & Guidance	CPA																																				
<p>Short multiplication of 3-digit number by 1-digit number</p> <p><i>To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.</i></p> <p><i>With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.</i></p> <p><i>This method and the language to use are best understood through the tutorial videos found here on the toolkit.</i></p>	<p>Exemplification of this process is best understood through viewing the video tutorial</p> <div style="display: flex; justify-content: space-between;"> <table border="1" data-bbox="727 409 1086 714"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>● ●</td> <td>● ● ● ● ●</td> <td>●</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <table border="1" data-bbox="727 748 1086 1052"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>● ●</td> <td>● ● ● ● ●</td> <td></td> </tr> <tr> <td></td> <td></td> <td>● ● ●</td> </tr> </tbody> </table> <table border="1" data-bbox="727 1086 1086 1391"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>● ●</td> <td></td> <td></td> </tr> <tr> <td>●</td> <td>● ●</td> <td>● ● ● ●</td> </tr> </tbody> </table> <table border="1" data-bbox="727 1424 1086 1729"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td>● ● ● ● ● ● ● ●</td> <td>● ●</td> <td>● ● ● ●</td> </tr> </tbody> </table> </div> <div style="margin-top: 20px;"> <p>To calculate 241×3, represent the number 241.</p> <p>Multiply each part by 3, regrouping as needed.</p> $\begin{array}{r} 241 \\ \times \quad 3 \\ \hline 723 \\ 1 \end{array}$ </div>	Hundreds	Tens	Ones	● ●	● ● ● ● ●	●				Hundreds	Tens	Ones	● ●	● ● ● ● ●				● ● ●	Hundreds	Tens	Ones	● ●			●	● ●	● ● ● ●	Hundreds	Tens	Ones				● ● ● ● ● ● ● ●	● ●	● ● ● ●
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Y4 Division

Strategies & Guidance	CPA																				
<p>Dividing by 10 and 100</p> <p>When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.</p> <p>When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. $210 \div 10 = 21$</p>	<div style="text-align: center;">CPA</div> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 25%;">thousands</th> <th style="width: 25%;">hundreds</th> <th style="width: 25%;">tens</th> <th style="width: 25%;">ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td></td> <td></td> <td style="text-align: center;"> 3</td> <td style="text-align: center;"> 3</td> </tr> <tr> <td style="text-align: center;"> 3</td> <td style="text-align: center;"> 3</td> <td style="text-align: center;">○</td> <td style="text-align: center;">○</td> </tr> <tr> <td style="text-align: center;">○</td> <td style="text-align: center;">○</td> <td style="text-align: center;">○</td> <td style="text-align: center;">○</td> </tr> </tbody> </table> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="width: 60%;"> <p>$30 \div 10 = 3$</p> <p>$300 \div 100 = 3$</p> <p>$3000 \div 1000 = 3$</p> <p>$300 \div 10 = 30$</p> <p>$3000 \div 100 = 30$</p> <p>$3000 \div 10 = 300$</p> </div> </div>	thousands	hundreds	tens	ones							 3	 3	 3	 3	○	○	○	○	○	○
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<p>Derived facts</p> <p>Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.</p> <p>Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.</p>	<div style="text-align: center; margin-bottom: 10px;"> $21 \div 3 = 7$  </div> <div style="text-align: center; margin-bottom: 10px;"> $21 \div 7 = 3$  </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="width: 45%;"> <p>$210 \div 7 = 30$</p> <p>$210 \div 3 = 70$</p> <p>$210 \div 30 = 7$</p> <p>$210 \div 70 = 3$</p> </div> <div style="width: 45%;"> <p>$2100 \div 7 = 300$</p> <p>$2100 \div 3 = 700$</p> <p>$2100 \div 300 = 7$</p> <p>$2100 \div 700 = 3$</p> </div> </div>																				

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<p>Short division of 4-digit numbers by 1-digit numbers</p> <p><i>Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.</i></p> <p><i>They progress to calculations that require regrouping in the hundreds or tens columns.</i></p> <p><i>Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.</i></p> <p><i>Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.</i></p>	<p>Division as sharing</p>  <p>Division as grouping</p> 
<p>Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths</p> <p><i>When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.</i></p>	<p>$24 \div 10 = 2.4$</p>  <p>$24 \div 100 = 0.24$</p> 